

YOUR NAME: \_\_\_\_\_

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Read each problem **very carefully** before starting to solve it. Each problem is worth 10 points. It is necessary to show **all** your work. Correct answers without explanations are worth 0 points. GOOD LUCK!!

1. Compute the following derivatives:

(a)  $\left[6\sqrt[3]{x^2} - \frac{9}{\sqrt[3]{x}}\right]' =$

(b)  $[x^3(2x - 3)^5]' =$

(c)  $\left[\frac{x + 1}{3x^2 - 7x}\right]' =$

2. Find an equation for the tangent line to  $f(x) = x^2(2x^3 + x + 5)^3$  at  $x = -1$ .

3. Suppose that an object moving on a straight line is located at  $s(t) = \frac{5t + 2}{t - 1}$  meters from the origin at time  $t > 1$  in seconds.

(a) Find the velocity of the object at  $t = 2$  seconds.

(b) Find its acceleration at time  $t = 3$  seconds.

4. Consider the function  $f(x) = \frac{x}{x^2 - 9}$ .

(a) Find the domain of  $f$ .

(b) Find the vertical and horizontal asymptotes of  $f$ .

Vertical Asymptotes (Lines):

Horizontal Asymptotes (Lines):

(c) Find the first derivative and its critical points.

(d) Create a sign table for the first derivative and draw conclusions about the monotonicity (intervals where it is increasing/decreasing) and the relative extrema (max/min) of  $f$ .

5. Consider the function  $f(x) = -x^4 + 4x^3$ .

(a) Find the first derivative and its critical points.

(b) Find the second derivative and its critical points.

(c) Create the **combined sign table** for the first and the second derivatives, making sure to give a summary of the monotonicity, relative extrema, concavity and inflection points of  $f$  at the last line of the table.

(d) Roughly sketch the graph of  $y = f(x)$ , clearly indicating (by labels) all important points.