

YOUR NAME: _____

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Read each problem **very carefully** before starting to solve it. Each problem is worth 10 points. It is necessary to show **all** your work. Correct answers without explanations are worth 0 points. GOOD LUCK!!

1. Find the slope of the tangent line to the graph of $16y^3 + 8xy^2 = x^2$ at the point $(x, y) = (4, -1)$.

2. A company produces certain electronic gadgets. The cost of each is \$50 and when x gadgets are produced they are sold at the price of $p(x) = 300 - 2x$ dollars/gadget.

- (a) The cost function is

$$C(x) =$$

- (b) The revenue function is

$$R(x) =$$

- (c) The profit function is

$$P(x) =$$

- (d) Suppose that the company raises production by 3 gadgets per week. Find the rate of change of the company's profit when 50 gadgets are produced.

3. Compute the derivatives:

(a) $\left(e^{x^5-3x^2}\right)' =$

(b) $\left(\sqrt[3]{x^7 + 5 \ln x}\right)' =$

(c) $\left(\frac{e^{3x} - e^{-2x}}{x^2 + e^{-x}}\right)' =$

4. An injection is administered to a patient and the concentration $C(t)$ of the active ingredient at time t (in hours since the injection) in the bloodstream is modeled by

$$C(t) = 3 + 7e^{-\frac{1}{5}t} \text{ units.}$$

- (a) What is the initial concentration of the medication in the bloodstream?
- (b) If the concentration of the ingredient must not be below 5 units for it to be effective, how long will it be before a new injection is needed?
- (c) What is the rate of change of the concentration 5 hours after the injection is given?
(Please provide appropriate unit with your answer.)

5. Compute the following integrals:

$$(a) \int \left(21\sqrt{x^5} + \frac{6}{\sqrt{x^5}} \right) dx =$$

$$(b) \int \frac{5x^7 - 3x^4 - 17x}{x} dx =$$