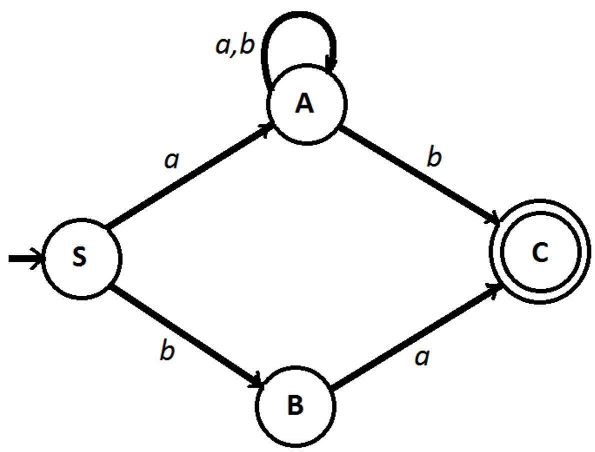
QUIZ 9 SOLUTIONS - CSCI 341 Friday, November 2

George Voutsadakis

Read each problem **very carefully** before starting to solve it. Each problem is worth 5 points. It is necessary to show **all** your work. Correct answers without explanations are worth 0 points. GOOD LUCK!!

1. Consider the regular language L given by the regular expression $a(a+b)^*b+ba$.

   (a) Create an NFA accepting the language L.



   (b) Use your NFA of Part (a) to construct a regular grammar G producing the language L. The regular grammar produced by our algorithm is

$$\begin{array}{rcl} S & \rightarrow & aA|bB \\ A & \rightarrow & aA|bA|bC \\ B & \rightarrow & aC \\ C & \rightarrow & \Lambda \end{array}$$

   This can be simplified as follows:

$$\begin{array}{rcl} S & \rightarrow & aA|ba \\ A & \rightarrow & aA|bA|b \end{array}$$

2. (a) Use the Pumping Lemma for regular languages to show that the language

$$L = \{a^n b^k : n, k \in \mathbb{N}, n \leq k\}$$

is not regular.

Suppose that L is regular. Then there exists an $m > 0$, such that, for all $s \in L$, with $|s| \geq m$, there exist $x, y, z \in \{a, b\}^*$, such that

$$s = xyz, \quad y \neq \Lambda, \quad |xy| \leq m \quad \text{and} \quad xy^k z \in L.$$

Now let $s = a^m b^m \in L$. Since $|s| = 2m \geq m$, there exist $x, y, z \in \{a, b\}^*$, such that

$$a^m b^m = xyz, \quad y \neq \Lambda, \quad |xy| \leq m, \quad xy^k z \in L.$$

Since $|xy| \leq m$ and $y \neq \Lambda$, we conclude $y = a^\ell$, for some $\ell > 0$.
But then, $xy^2 z = a^{m+\ell} b^m \notin L$, contradicting the Pumping Lemma.

(b) Use Part (a) and closure properties of regular languages to prove that the language $M = \{a^n b^k : n, k \in \mathbb{N}, n > k\}$ is not regular.

Suppose, for the sake of obtaining a contradiction, that M is regular.
Since the complements of regular languages are regular, we get that

$$M' = \{a, b\}^* - M$$

is regular.
Since, obviously, $a^* b^*$ is regular and the intersection of two regular languages is regular, we get that

$$a^* b^* \cap M'$$

is also regular.
But which is this language? A moment's thought shows that

$$a^* b^* \cap M' = \{a^n b^k : n, k \in \mathbb{N}, n \leq k\} = L,$$

which was shown to be non-regular in Part (a)!
Therefore M cannot be regular.