Read each problem **very carefully** before starting to solve it. Each problem is worth 5 points. It is necessary to show **all** your work. Correct answers without explanations are worth 0 points. GOOD LUCK!!

1. Explain using a diagram and a short, but precise, sentence why, for any angle  $\theta$ ,  $\sin^2 \theta + \cos^2 \theta = 1$ .

2. Use the identity of the previous part to show that, for any angle  $\theta$  for which  $\sin \theta \neq 0$ ,  $1 + \cot^2 \theta = \csc^2 \theta$ . Please, explain all your steps.

3. Use the method of the conjugate to show that, for any angle  $\theta$ ,  $\frac{\sin \theta}{1 + \cos \theta} = \frac{1 - \cos \theta}{\sin \theta}$ . Please, explain all your steps.