Read each problem **very carefully** before starting to solve it. Each problem is worth 10 points. It is necessary to show **all** your work. Correct answers without explanations are worth 0 points. GOOD LUCK!!

1. (a) Use transformations to sketch the graph of the function $g(x) = -(x-3)^2 + 2$. Explain the process.

   (b) Sketch the graph of the piece-wise defined function

   $$f(x) = \begin{cases} x, & \text{if } x < 2 \\ -(x-3)^2 + 2, & \text{if } x \geq 2 \end{cases}$$

   (c) Find the following values concerning the function f of the previous part:

   $$\lim_{x \to 2^-} f(x) =$$
   $$\lim_{x \to 2^+} f(x) =$$
   $$\lim_{x \to 2} f(x) =$$

2. Calculate the following limits:

   (a)

   $$\lim_{x \to -3} \frac{x^2 - 9}{2x^2 + 7x + 3}$$

   (b)

   $$\lim_{x \to 5} \frac{x - 5}{\sqrt{x + 11} - 4}$$

3. Calculate the limit

$$\lim_{\theta \to 0} \frac{\sin \theta}{\theta + \tan \theta}$$

4. (a) Graph the piece-wise defined function

$$f(x) = \begin{cases} 1 - x^2, & \text{if } x < 1 \\ \frac{1}{x}, & \text{if } x \geq 1 \end{cases}$$

(b) Find the following values:

$$\lim_{x \to 1^-} f(x) =$$
$$\lim_{x \to 1^+} f(x) =$$
$$f(1) =$$

(c) Is f continuous at $x = 1$? Please explain clearly.

5. Consider the following function, where c is a constant representing a fixed real number:

$$f(x) = \begin{cases} cx^2 + 2x, & \text{if } x < 2 \\ x^3 - cx, & \text{if } x \geq 2 \end{cases}$$

(a) Find the following values:

$$f(2) \quad =$$

$$\lim_{x \to 2^-} f(x) \quad =$$

$$\lim_{x \to 2^+} f(x) \quad =$$

(b) For what value of the real constant c is the function f continuous at $x = 2$? Please, show and briefly explain your work.