

YOUR NAME: \_\_\_\_\_

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Read each problem **very carefully** before starting to solve it. Each problem is worth 10 points. It is necessary to show **all** your work. Correct answers without explanations are worth 0 points. GOOD LUCK!!

1. Consider the function  $f(x) = x^4 - 4x^3$ . Create the relevant sign table (including signs of first and second derivative), show the intervals of monotonicity and concavity, the relative extrema and the inflection points and sketch its graph, clearly depicting all critical points.

2. Consider the function  $f(x) = \frac{1}{1+x^2}$ . Create the relevant sign table (including signs of first and second derivative), show the intervals of monotonicity and concavity, the relative extrema and the inflection points. You **do not need** to sketch the graph for this problem.

3. A motorbike shop estimates that it will sell 600 bikes in a year. Each bike costs \$ 300, plus a fixed charge of \$ 200 per order. If it costs \$ 60 to store a bike for a year, what is the order size that are needed to minimize cost?
4. Find an equation for the tangent line to the graph of  $x^2y^2 - xy = 2$  at the point  $(-1, 1)$ .
5. A large spherical balloon is being inflated at the rate of  $32 \text{ ft}^3$  per minute. How fast is its radius increasing at the moment when its radius is 2 feet? (Recall that the volume of a sphere is given by  $V = \frac{4}{3}\pi r^3$ , where  $r$  is the length of its radius.)