Read each problem **very carefully** before starting to solve it. Each problem is worth 5 points. It is necessary to show **all** your work. Correct answers without explanations are worth 0 points. GOOD LUCK!!

1. Follow instructions very closely: Consider the quadratic function $f(x) = -x^2 + 2x + 3$.

   (a) Find the vertex of the parabola $y = f(x)$.

   (b) State the opening direction and explain.

   (c) Find the x- and y-intercepts of $y = f(x)$.

   (d) Roughly sketch the graph of $y = f(x)$ indicating clearly all important points.

2. Let $f(x) = -x^2 + 2x + 3$ (the quadratic function of the previous problem). Consider the following piece-wise defined function:

$$g(x) = \begin{cases} f(x), & \text{if } x < 2 \\ 1, & \text{if } x = 2 \\ x - 2, & \text{if } x > 2 \end{cases}$$

   (a) Sketch the graph of $y = g(x)$.

   (b) Using the graph, find the following limits:

   $$\lim_{x \to 2^-} g(x) \quad =$$

   $$\lim_{x \to 2^+} g(x) \quad =$$