

YOUR NAME: _____

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Read each problem **very carefully** before starting to solve it. Each problem is worth 10 points. It is necessary to show **all** your work. Correct answers without explanations are worth 0 points. GOOD LUCK!!

1. Find the radius and the interval of convergence of the power series $\sum_{n=0}^{\infty} \left(\frac{3}{4}\right)^n (x+5)^n$.
2. Find a power series for $f(x) = \ln(1+x)$. Using the power series for $f(x)$, find the sum of the alternating harmonic series (recall that we have shown, using the alternating series test, that it converges, but we did not find its sum).

3. Use the \sum -notation to write the Taylor series of the function $f(x) = \sin \pi x$ around $a = \frac{1}{2}$.

4. The parametric curve $\begin{cases} x &= 2 - \pi \cos t \\ y &= 2t - \pi \sin t \end{cases}$, $-\pi \leq t \leq \pi$, crosses itself at a point on the positive x -axis. Find equations for the two tangent lines at that point.

5. An ellipse centered at the origin with vertices at $(-a, 0)$, $(a, 0)$ and $(0, b)$, $(0, -b)$ is given by the parametric equations $\begin{cases} x &= a \cos \theta \\ y &= b \sin \theta \end{cases}$, $0 \leq \theta \leq 2\pi$. Find the area that is enclosed by the ellipse.