EXAM 1 - MATH 251 Friday, February 1

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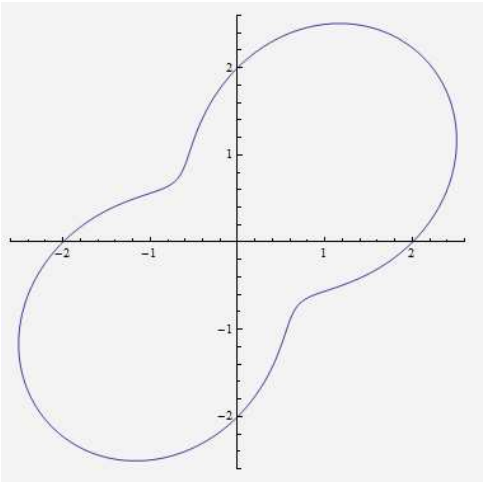
Read each problem **very carefully** before starting to solve it. Each problem is worth 10 points. It is necessary to show **all** your work. Correct answers without explanations are worth 0 points. GOOD LUCK!!

1. Find the length of the path $\begin{cases} x &= \sin\theta - \theta\cos\theta \\ y &= \cos\theta + \theta\sin\theta \end{cases}, 0 \leq \theta \leq 2.$

2. Find the area enclosed by the polar curve $r(\theta) = 2 + \sin 2\theta$.



3. Identify the conic and determine its center, vertices, foci (or focus and directrix)

$$4x^2 - 25y^2 + 16x + 50y - 109 = 0.$$

4. Consider the points $P(-3, 2, -7)$ and $Q(-2, 5, -2)$ in 3-dimensional space.

   (a) Find an equation for the sphere with center P and passing through Q.

   (b) Find a unit vector in the direction of $\overrightarrow{PQ}$.

   (c) Are the vectors $\overrightarrow{PQ}$ and $\mathbf{v} = \langle -3, 2, -\frac{1}{2} \rangle$ perpendicular? Explain your work.

5. Calculate the cosines of the following angles:

   (a) The angle between a diagonal of a cube and one of its edges;

   (b) The angle between a diagonal of a cube and a diagonal of one of its faces.