HOMEWORK 2 SOLUTIONS - MATH 300
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**Problem 1** *Consider the valuation* $\mathbf{e} = (1, 0, 0, 1)$ *for the propositional variables* $P, Q, R, S$. *For each of the following formulas* $F$, *show* $F(\mathbf{e})$ *and find* $\widehat{F}(\mathbf{e})$.

(a) $R \to (S \wedge P)$;

(b) $P \to (R \to S)$;

(c) $(P \vee R) \leftrightarrow (R \wedge \neg S)$;

(d) $(Q \wedge \neg S) \to (P \leftrightarrow S)$;

(e) $R \wedge S \to (P \to \neg Q \vee S)$;

(f) $(P \vee \neg Q) \vee R \to (S \wedge \neg S)$.

**Solution:**

| Formla $F$ | $F(\mathbf{e})$ | $\widehat{F}(\mathbf{e})$ |
|---|---|---|
| $R \to (S \wedge P)$ | $0 \to (1 \wedge 1)$ | 1 |
| $P \to (R \to S)$ | $1 \to (0 \to 1)$ | 1 |
| $(P \vee R) \leftrightarrow (R \wedge \neg S)$ | $(1 \vee 0) \leftrightarrow (0 \wedge \neg 1)$ | 0 |
| $(Q \wedge \neg S) \to (P \leftrightarrow S)$ | $(0 \wedge \neg 1) \to (1 \leftrightarrow 1)$ | 1 |
| $R \wedge S \to (P \to \neg Q \vee S)$ | $0 \wedge 1 \to (1 \to \neg 0 \vee 1)$ | 1 |
| $(P \vee \neg Q) \vee R \to (S \wedge \neg S)$ | $(1 \vee \neg 0) \vee 0 \to (1 \wedge \neg 1)$ | 0 |

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**Problem 2** *Construct truth tables for each of the following formulas:*

(a) $P \to (P \to Q)$;

(b) $P \to \neg(Q \wedge R)$;

(c) $(P \to Q) \leftrightarrow \neg P \vee Q$;

(d) $P \wedge Q \to (Q \wedge \neg Q \to R \wedge Q)$.

**Solution:**

(a)

| $P$ | $Q$ | $P \to Q$ | $P \to (P \to Q)$ |
|---|---|---|---|
| 0 | 0 | 1 | 1 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 |
| 1 | 1 | 1 | 1 |

(b)

| $P$ | $Q$ | $R$ | $Q \wedge R$ | $\neg(Q \wedge R)$ | $P \to \neg(Q \wedge R)$ |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 1 | 1 |
| 0 | 0 | 1 | 0 | 1 | 1 |
| 0 | 1 | 0 | 0 | 1 | 1 |
| 0 | 1 | 1 | 1 | 0 | 1 |
| 1 | 0 | 0 | 0 | 1 | 1 |
| 1 | 0 | 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 0 | 1 | 1 |
| 1 | 1 | 1 | 1 | 0 | 0 |

(c)

| $P$ | $Q$ | $P \to Q$ | $\neg P$ | $\neg P \vee Q$ | $(P \to Q) \leftrightarrow \neg P \vee Q$ |
|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 | 0 | 1 |
| 1 | 1 | 1 | 0 | 1 | 1 |

(d) One can show very easily that the formula

$$P \wedge Q \to (Q \wedge \neg Q \to R \wedge Q)$$

is a tautology! In fact, $Q \wedge \neg Q$ is always evaluated to 0; thus, $Q \wedge \neg Q \to R \wedge Q$ is always evaluated to 1 and, therefore, $P \wedge Q \to (Q \wedge \neg Q \to R \wedge Q)$ is always evaluated to 1 also!

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**Problem 3** *For each of the following formulas determine whether the information given is sufficient to decide its truth value. If it is, state that truth value; if it is not, show that both truth values are possible.*

(a) $(P \to Q) \to R$; *$R$ is assigned truth value* 1;

(b) $P \vee (Q \to R)$; *truth value of* $Q \to R$ *is* 1;

(c) $(P \to Q) \to (\neg Q \to \neg P)$; *$Q$ is assigned truth value* 1;

(d) $(P \wedge Q) \to (P \vee S)$; *$P$ is assigned truth value* 1 *and $S$ truth value* 0;

**Solution:**

(a) Since $R$ is assigned truth value 1 and any implication whose conclusion is true is also true, $(P \to Q) \to R$ is also evaluated to 1.

(b) Since the truth value of $Q \to R$ is 1 and any disjunction one of whose disjuncts has truth value 1 is also evaluated to 1, we get that $P \vee (Q \to R)$ is evaluated to 1.

(c) Since $Q$ is assigned truth value 1, the implication $P \to Q$ is evaluated to 1. Moreover $\neg Q$ is evaluated to 0, whence the implication $\neg Q \to \neg P$ is also evaluated to 1. Therefore, both the premiss and the conclusion of the implication $(P \to Q) \to (\neg Q \to \neg P)$ are assigned truth value 1, showing that the implication $(P \to Q) \to (\neg Q \to \neg P)$ is also evaluated to 1.

(d) Since $P$ is assigned truth value 1, the conclusion of $(P \wedge Q) \to (P \vee S)$ is evaluated to 1, whence the implication $(P \wedge Q) \to (P \vee S)$ is also evaluated to 1.

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**Problem 4** *The following statement*

> *"If labor or management is stubborn, then the strike will be settled if and only if the government obtains an injunction, but troops are not sent into the factory"*

*may be expressed as a formula* $L \vee M \to (S \leftrightarrow G \wedge \neg R)$, *with the obvious implied meaning for the propositional variables involved.*

(a) *Determine the truth value of the given statement under the assumption that both labor and management are stubborn, the strike will not be settled, the government fails to obtain an injunction and troops are sent into the factory.*

(b) *Determine the truth value of the given statement if it is agreed that*

> *"If the government obtains an injunction, then troops will be sent into the factory. If troops are sent into the factory, then the strike will not be settled. The strike will be settled. The management is stubborn."*

**Solution:**

(a) We have

| | $L$ | $\vee$ | $M$ | $\rightarrow$ | ( | $S$ | $\leftrightarrow$ | $G$ | $\wedge$ | $\neg$ | $R$ | ) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Assignment | 1 | | 1 | | | 0 | | 0 | | | 1 | |
| | | 1 | | | | 0 | | | 0 | | | |
| | | 1 | | | | | 1 | | | | | |
| Evaluation | | | | 1 | | | | | | | | |

(b) Since the boxed statements are assumed to be true, we get an evaluation that assigns:

| Formula | $G \rightarrow R$ | $R \rightarrow \neg S$ | $S$ | $M$ |
|---|---|---|---|---|
| Evaluation | 1 | 1 | 1 | 1 |

Since $\neg S$ is evaluated to 0 and $R \rightarrow \neg S$ is evaluated to 1, $R$ must be evaluated to 0. Similarly, since $R$ is evaluated to 0 and $G \rightarrow R$ is evaluated to 1, $G$ must also be evaluated to 0. Now, we have gathered enough information: Knowing that $G, R$ have truth values 0 and $S, M$ have truth values 1, we see that $L \vee M \rightarrow (S \leftrightarrow G \wedge \neg R)$ is evaluated to 0.

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**Problem 5** *Which pairs of the following propositional formulas are truth equivalent? (Show all work.)*

(a) $\neg(P \leftrightarrow (R \leftrightarrow P))$

(b) $P \vee ((P \leftrightarrow R) \vee P)$

(c) $R \vee ((\neg Q \leftrightarrow P) \leftrightarrow Q)$

(d) $(R \rightarrow (\neg P \rightarrow P)) \vee P$

(e) $(R \leftrightarrow P) \vee ((P \vee (Q \vee R)) \rightarrow P)$

**Solution:**

| $P$ | $Q$ | $R$ | (a) | (b) | (c) | (d) | (e) |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 |
| 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 |
| 0 | 1 | 0 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 0 | 0 | 1 | 0 | 0 |
| 1 | 0 | 0 | 1 | 1 | 0 | 1 | 1 |
| 1 | 0 | 1 | 0 | 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 1 | 1 | 0 | 1 | 1 |
| 1 | 1 | 1 | 0 | 1 | 1 | 1 | 1 |

From the combined truth table, we can see that the formulas $P \vee ((P \leftrightarrow R) \vee P), (R \rightarrow (\neg P \rightarrow P)) \vee P$ and $(R \leftrightarrow P) \vee ((P \vee (Q \vee R)) \rightarrow P)$ ((b), (d) and (e)) are truth equivalent. ■

**Problem 6** *Which of the following propositional formulas are tautologies and which contradictions? (Show all work.)*

(a) $((P \leftrightarrow ((\neg Q) \vee R)) \rightarrow ((\neg P) \rightarrow Q))$

(b) $((P \to (Q \vee R)) \vee (P \to Q))$

(c) $((P \leftrightarrow Q) \leftrightarrow (P \leftrightarrow (Q \leftrightarrow P)))$

(d) $((P \vee (\neg(Q \wedge R))) \to ((P \leftrightarrow R) \vee Q))$

(e) $(\neg((\neg R) \to ((S \wedge Q) \to S)))$

**Solution:**

(a)

| $P$ | $Q$ | $R$ | $\neg Q$ | $\neg P$ | $\neg Q \vee R$ | $P \leftrightarrow \neg Q \vee R$ | $\neg P \to Q$ | $(P \leftrightarrow \neg Q \vee R) \to (\neg P \to Q)$ |
|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 1 |
| 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 1 |
| 0 | 1 | 0 | 0 | 1 | 0 | 1 | 1 | 1 |
| 0 | 1 | 1 | 0 | 1 | 1 | 0 | 1 | 1 |
| 1 | 0 | 0 | 1 | 0 | 1 | 1 | 1 | 1 |
| 1 | 0 | 1 | 1 | 0 | 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 1 |
| 1 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 1 |

Thus, $((P \leftrightarrow ((\neg Q) \vee R)) \to ((\neg P) \to Q))$ is a tautology.

(b)

| $P$ | $Q$ | $R$ | $Q \vee R$ | $P \to Q \vee R$ | $P \to Q$ | $(P \to Q \vee R) \vee (P \to Q)$ |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 1 | 1 | 1 |
| 0 | 0 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 0 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 1 | 1 | 0 | 1 |
| 1 | 1 | 0 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 |

Therefore, $((P \to (Q \vee R)) \vee (P \to Q))$ is neither a tautology nor a contradiction.

(c)

| $P$ | $Q$ | $P \leftrightarrow Q$ | $Q \leftrightarrow P$ | $P \leftrightarrow (Q \leftrightarrow P)$ | $(P \leftrightarrow Q) \leftrightarrow (P \leftrightarrow (Q \leftrightarrow P))$ |
|---|---|---|---|---|---|
| 0 | 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 0 | 0 | 1 | 0 |
| 1 | 0 | 0 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 |

Therefore, $((P \to (Q \vee R)) \vee (P \to Q))$ is neither a tautology nor a contradiction.

(d)

| $P$ | $Q$ | $R$ | $Q \wedge R$ | $\neg(Q \wedge R)$ | $P \leftrightarrow R$ | $P \vee \neg(Q \wedge R)$ | $(P \leftrightarrow R) \vee Q$ | $P \vee \neg(Q \wedge R) \to (P \leftrightarrow R) \vee Q$ |
|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 |
| 0 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 0 |
| 0 | 1 | 0 | 0 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 0 | 0 | 0 | 1 | 1 |
| 1 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 |
| 1 | 0 | 1 | 0 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 0 | 1 | 0 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 1 |

Therefore, $((P \vee (\neg(Q \wedge R))) \to ((P \leftrightarrow R) \vee Q))$ is neither a tautology nor a contradiction.

(e)

| $Q$ | $R$ | $S$ | $S \wedge Q$ | $\neg R$ | $S \wedge Q \to S$ | $\neg R \to (S \wedge Q \to S)$ | $\neg(\neg R \to (S \wedge Q \to S))$ |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 0 | 1 | 1 | 1 | 0 |
| 0 | 1 | 0 | 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 | 1 | 1 | 1 | 0 |
| 1 | 0 | 1 | 1 | 1 | 1 | 1 | 0 |
| 1 | 1 | 0 | 0 | 0 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 0 | 1 | 1 | 0 |

Thus $(\neg((\neg R) \to ((S \wedge Q) \to S)))$ is a contradiction.

∎

**Problem 7** *Let $F, G$ be propositional formulas. Show that, if the formulas $F$ and $(F \to G)$ are tautologies, then so is $G$.*

**Solution:** Let $\mathbf{e}$ be a (arbitrary) truth assignment. Since $F$ and $(F \to G)$ are tautologies, we must have $\hat{F}(\mathbf{e}) = 1$ and $\widehat{(F \to G)}(\mathbf{e}) = \widehat{F}(\mathbf{e}) \to \widehat{G}(\mathbf{e}) = 1 \to \widehat{G}(\mathbf{e}) = 1$. Thus, by the truth table of $\to$, we must have $\widehat{G}(\mathbf{e}) = 1$. Since $\mathbf{e}$ was arbitrary, this shows that $G$ is always evaluated to 1, i.e., it must be a tautology. ∎

**Problem 8**

*Show that the set of connectives $\{\neg, \leftrightarrow\}$ is not adequate.* (**Hint:** *Study the work we did with $\to$ in class!*)

**Solution:** We show that every formula $F(P, Q)$ which uses only the connectives $\neg$ and $\leftrightarrow$ is evaluated to 1 at an even number of the four possible truth assignments. This clearly implies that it is not possible to express either $\vee$ or $\wedge$ using only the connectives in $\{\neg, \leftrightarrow\}$.

**Base of Structural Induction:** For the base case, note that $F(P, Q) = P$ is evaluated to 1 at two of the four possible truth assignments, namely those where $P$ is assigned the truth value 1.

**Structural Induction Hypothesis:** Assume, next, that $G(P, Q)$ and $H(P, Q)$ are two formulas in the propositional variables $P$ and $Q$, containing only connectives from $\{\neg, \leftrightarrow\}$, that are evaluated to 1 at an even number of truth assignments.

**Step of Structural Induction:** There are two cases to consider in this step:

- Case 1: If $F(P, Q) = (\neg G(P, Q))$, then $F(P, Q)$ is evaluated to 1 at all truth assignments at which $G(P, Q)$ is evaluated to 0 and their number, being, by the induction hypothesis, four minus an even number, is also an even number.

- Case 2: If $F(P, Q) = (G(P, Q) \leftrightarrow H(P, Q))$, then $F(P, Q)$ is evaluated to 1 at all truth assignments at which $G(P, Q)$ and $H(P, Q)$ are evaluated to the same truth value. We show that this number is actually even! Define

$$\begin{aligned} N_{00} &= \text{Number of truth assignments at which both } G \text{ and } H \text{ are evaluated to 0} \\ N_{01} &= \text{Number of truth assignments at which } G \text{ is evaluated to 0 and } H \text{ to 1} \\ N_{10} &= \text{Number of truth assignments at which } G \text{ is evaluated to 1 and } H \text{ to 0} \\ N_{11} &= \text{Number of truth assignments at which both } G \text{ and } H \text{ are evaluated to 1} \end{aligned}$$

  Then, by the induction hypothesis, $N_{00}+N_{01}$ (the number of assignments where $G$ is evaluated to 0) is even, and $N_{01} + N_{11}$ (the number of assignments at which $H$ is evaluated to 1) is also even. Therefore, their sum $N_{00} + N_{01} + N_{01} + N_{11}$ is even. This shows that $N_{00} + N_{11}$ is an even number minus $2N_{01}$, i.e., is also even. But this is exactly the number of assignments at which $G(P, Q)$ and $H(P, Q)$ are evaluated to the same truth value!

∎

**Problem 9** *In class, we showed that $\{\curlywedge\}$ and $\{|\}$ are adequate sets of connectives. Express the propositional formula $P \wedge (Q \to R)$ in terms of $|$. (Please, do not just write a single formula; provide a couple of steps with explanation on the way!)*

**Solution:**
**First of Many Possible Solutions:**

$$\begin{aligned} P \wedge (Q \to R) &\sim P \wedge (\neg Q \vee R) \quad (Q \to R \sim \neg Q \vee R) \\ &\sim (P \wedge \neg Q) \vee (P \wedge R) \quad \text{(distributivity)} \\ &\sim \neg(\neg P \vee Q) \vee \neg(\neg P \vee \neg R) \quad \text{(De Morgan's Laws)} \\ &\sim \neg(P|(\neg Q)) \vee \neg(P|R) \quad (\text{since } P|Q \sim \neg P \vee \neg Q) \\ &\sim \neg(P|(Q|Q)) \vee \neg(P|R) \quad (\text{since } \neg P \sim P|P) \\ &\sim (P|(Q|Q))|(P|R) \quad (\text{since } P|Q \sim \neg P \vee \neg Q) \end{aligned}$$

**Second of Many Possible Solutions:**
Note

$$P \to Q \sim \neg P \vee Q \sim P|(\neg Q) \sim P|(Q|Q). \tag{1}$$

and, also,

$$P \wedge Q \sim \neg(\neg P \vee \neg Q) \sim \neg(P|Q) \sim (P|Q)|(P|Q). \tag{2}$$

$$\begin{aligned} P \wedge (Q \to R) &\sim P \wedge (Q|(R|R)) \quad \text{(by Truth-Equivalence (1))} \\ &\sim (P|(Q|(R|R)))|(P|(Q|(R|R))) \quad \text{(by Truth-Equivalence (2)).} \end{aligned}$$

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**Problem 10** *Find the disjunctive normal forms, first using rewriting rules (the $\rightsquigarrow$-rules of the slides), and, then, using the method of truth tables.*

(a) $(P \to Q) \to (Q \to R)$

(b) $Q \to (Q \to (R \to S))$

**Solution:**

(a) First, we the $\rightsquigarrow$-rules:

$$\begin{aligned} &(\neg P \vee Q) \to (\neg Q \vee R) \\ \rightsquigarrow\ &\neg(\neg P \vee Q) \vee (\neg Q \vee R) \\ \rightsquigarrow\ &(P \wedge \neg Q) \vee \neg Q \vee R \\ \rightsquigarrow\ &\neg Q \vee R \\ \rightsquigarrow\ &(\neg P \wedge \neg Q \wedge \neg R) \vee (\neg P \wedge \neg Q \wedge R) \vee (P \wedge \neg Q \wedge \neg R) \vee (P \wedge \neg Q \wedge R) \\ &\vee (\neg P \wedge \neg Q \wedge R) \vee (\neg P \wedge Q \wedge R) \vee (P \wedge \neg Q \wedge R) \vee (P \wedge Q \wedge R) \\ \rightsquigarrow\ &(\neg P \wedge \neg Q \wedge \neg R) \vee (\neg P \wedge \neg Q \wedge R) \vee (\neg P \wedge Q \wedge R) \vee (P \wedge \neg Q \wedge \neg R) \\ &\vee (P \wedge \neg Q \wedge R) \vee (P \wedge Q \wedge R). \end{aligned}$$

Next, with the truth table:

| $P$ | $Q$ | $R$ | $P \to Q$ | $Q \to R$ | $(P \to Q) \to (Q \to R)$ |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 1 | 1 | 1 |
| 0 | 0 | 1 | 1 | 1 | 1 |
| 0 | 1 | 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 | 1 | 1 |
| 1 | 0 | 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 |

The last column of the truth table also results in the disjunctive normal form

$$(\neg P \wedge \neg Q \wedge \neg R) \vee (\neg P \wedge \neg Q \wedge R) \vee (\neg P \wedge Q \wedge R) \vee (P \wedge \neg Q \wedge \neg R) \vee (P \wedge \neg Q \wedge R) \vee (P \wedge Q \wedge R).$$

(b) First, we the $\rightsquigarrow$-rules:

$$
\begin{array}{ll}
\multicolumn{2}{l}{Q \to (Q \to (R \to S))} \\
\rightsquigarrow & Q \to (Q \to (\neg R \lor S)) \\
\rightsquigarrow & Q \to (\neg Q \lor (\neg R \lor S)) \\
\rightsquigarrow & \neg Q \lor \neg Q \lor \neg R \lor S \\
\rightsquigarrow & \neg Q \lor \neg R \lor S \\
\rightsquigarrow & \text{each expands to eight DNF-constituents, some common} \\
\rightsquigarrow & (\neg P \land \neg Q \land \neg R \land \neg S) \lor (\neg P \land \neg Q \land \neg R \land \neg S) \lor (\neg P \land \neg Q \land \neg R \land \neg S) \\
& \lor (\neg P \land \neg Q \land \neg R \land \neg S) \lor (\neg P \land \neg Q \land \neg R \land \neg S) \lor (\neg P \land \neg Q \land \neg R \land \neg S) \\
& \lor (\neg P \land \neg Q \land \neg R \land \neg S) \lor (\neg P \land \neg Q \land \neg R \land \neg S) \lor (\neg P \land \neg Q \land \neg R \land \neg S) \\
& \lor (\neg P \land \neg Q \land \neg R \land \neg S) \lor (\neg P \land \neg Q \land \neg R \land \neg S) \lor (\neg P \land \neg Q \land \neg R \land \neg S) \\
& \lor (\neg P \land \neg Q \land \neg R \land \neg S) \lor (\neg P \land \neg Q \land \neg R \land \neg S).
\end{array}
$$

Next, with the truth table:

| $P$ | $Q$ | $R$ | $S$ | $R \to S$ | $Q \to (R \to S)$ | $Q \to (Q \to (R \to S))$ |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 1 | 1 | 1 |
| 0 | 0 | 0 | 1 | 1 | 1 | 1 |
| 0 | 0 | 1 | 0 | 0 | 1 | 1 |
| 0 | 0 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 0 | 1 | 1 | 1 | 1 |
| 0 | 1 | 0 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 | 1 | 1 | 1 |
| 1 | 0 | 1 | 0 | 0 | 1 | 1 |
| 1 | 0 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 1 | 1 | 1 | 1 |
| 1 | 1 | 0 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 |

The last column of the truth table also results in the disjunctive normal form

$$
\begin{aligned}
&(\neg P \land \neg Q \land \neg R \land \neg S) \lor (\neg P \land \neg Q \land \neg R \land \neg S) \lor (\neg P \land \neg Q \land \neg R \land \neg S) \lor (\neg P \land \neg Q \land \neg R \land \neg S) \\
&\quad \lor (\neg P \land \neg Q \land \neg R \land \neg S) \lor (\neg P \land \neg Q \land \neg R \land \neg S) \lor (\neg P \land \neg Q \land \neg R \land \neg S) \lor (\neg P \land \neg Q \land \neg R \land \neg S) \\
&\quad \lor (\neg P \land \neg Q \land \neg R \land \neg S) \lor (\neg P \land \neg Q \land \neg R \land \neg S) \lor (\neg P \land \neg Q \land \neg R \land \neg S) \lor (\neg P \land \neg Q \land \neg R \land \neg S) \\
&\quad \lor (\neg P \land \neg Q \land \neg R \land \neg S) \lor (\neg P \land \neg Q \land \neg R \land \neg S).
\end{aligned}
$$

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