HOMEWORK 3 - MATH 300 — Due: Friday, February 22

YOUR NAME:______________________ — George Voutsadakis

Read each problem **very carefully** and try to understand it well before starting to solve it. It is necessary to show **all** your work. Correct answers without explanations are worth 0 points. Write your own solutions and be neat!! **Take pride in your work!! Do not hand in scratchy doodles.**

1. The questions of Problem 1 refer to the following combined truth table:

Line	P	Q	R	S	F_1	F_2	F_3	F_4	F_5	F_6	F_7	F_8	F_9	F_{10}	F_{11}	F_{12}
1	1	1	1	1	0	1	0	1	0	0	1	0	0	1	1	1
2	1	1	1	0	0	1	1	1	1	1	1	0	0	1	1	1
3	1	1	0	1	0	0	0	1	0	0	0	0	0	1	1	0
4	1	1	0	0	0	0	1	1	1	1	0	0	0	1	1	0
5	1	0	1	1	1	0	0	0	0	1	0	1	0	1	1	0
6	1	0	1	0	1	0	1	0	0	1	0	1	0	1	1	0
7	1	0	0	1	1	0	0	1	0	1	0	1	0	1	1	0
8	1	0	0	0	1	0	1	1	0	1	0	0	0	1	1	0
9	0	1	1	1	0	0	1	0	1	0	0	0	0	0	1	0
10	0	1	1	0	0	0	0	0	0	1	0	0	0	1	1	0
11	0	1	0	1	0	0	1	1	1	0	0	0	0	0	1	0
12	0	1	0	0	0	0	1	1	0	1	0	0	0	1	1	0
13	0	0	1	1	1	0	1	0	1	1	0	0	0	1	1	0
14	0	0	1	0	0	0	0	0	0	1	0	0	0	1	1	0
15	0	0	0	1	0	0	1	1	0	1	0	0	0	0	1	0
16	0	0	0	0	0	0	1	1	0	1	0	0	0	1	1	0

   (a) Which of the formulas F_1-F_{12} are truth equivalent?

   (b) Which of the formulas F_1-F_{12} are tautologies?

   (c) Which of the formulas F_1-F_{12} are contradictions?

   (d) Determine if the following arguments are valid. If not, cite the number of a row of the truth table that refutes the argument.

      (i) $F_1, F_5, F_{10} \therefore F_8$

      (ii) $F_1, F_3, F_4, F_5, F_6, F_7, F_{10} \therefore F_8$

   (e) Determine if the following collections of formulas are satisfiable. If so, cite the number of a row of the truth table that satisfies them.

      (i) $\{F_1, F_5, F_6\}$

      (ii) $\{F_3, F_4, F_6, F_8\}$

   (f) Find the disjunctive normal form of F_1 (with respect to the variables P, Q, R, S).

   (g) Find the conjunctive normal form of F_{10} (with respect to the variables P, Q, R, S).

2. Consider the following propositions:

A	"is able to stir the hearts of men"
C	"is clever"
S	"is Shakespeare"
P	"is a true poet"
N	"understands human nature"
H	"is the writer of Hamlet".

Express the following as an argument in the propositional calculus and determine if it is valid, where the universe of discourse is the class of all writers:

All writers who understand human nature are clever.
No writer is a true poet unless he can stir the hearts of men.
Shakespeare wrote Hamlet.
No writer who does not understand human nature can stir the hearts of men.
None but a true poet could have written Hamlet.
$\therefore$ Shakespeare is clever.

3. Determine if the following argument is a valid argument:

$$\begin{array}{l} \neg C \wedge D \\ \neg(\neg B \wedge C \wedge D) \\ \neg(\neg B \vee (\neg A \wedge B)) \wedge \neg C \wedge \neg D \\ \therefore\ A \wedge \neg B. \end{array}$$

4. Let $\mathcal{S}$ be an arbitrary infinite set of natural numbers, presented in binary notation (e.g., 12 is presented as 1100). Prove that there is an infinite sequence of different binary numbers $b_1, b_2, \ldots$, such that each b_i is a prefix of b_{i+1} and also a prefix of some element of $\mathcal{S}$.

5. (Generalization of the Erdös-deBruijn Theorem) In this problem, the following definition is needed:

A **homomorphism** f from a graph **G** to a graph **H** is a map $f : G \to H$, such that, if a and b are adjacent in **G**, then $f(a)$ and $f(b)$ are adjacent in **H**, i.e., such that f **preserves the adjacency relation**, or f **preserves edges**.

If **H** is a finite graph, prove that there is a homomorphism from **G** to **H** iff for every finite subgraph $\mathbf{G}_0$ of **G**, there is a homomorphism from $\mathbf{G}_0$ to **H**.

(**Caution:** In *graph theory*, what we call here a subgraph is usually referred to as an **induced subgraph**; In *logic* the usage accords with the concept of a **substructure** of an arbitrary first-order structure.)