

YOUR NAME: _____

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Read each problem **very carefully** before starting to solve it. Each problem is worth 5 points. It is necessary to show **all** your work. Correct answers without explanations are worth 0 points. GOOD LUCK!!

1. A car rental business finds that it can rent 60 cars if it charges \$80 for a weekend. It estimates that for each \$5 price increase it will rent three fewer cars. Moreover, the business has to pay \$30 per car for maintenance and has fixed costs \$1,800 every weekend. The goal is to find the price that should be charged to maximize the business' profit.

Assume x is the number of \$5 dollar increases. Find the following quantities:

The price charged $p(x) =$

The number of cars rented $q(x) =$

The Revenue function $R(x) =$

The Cost function $C(x) =$

The Profit function $P(x) =$

Use the profit function to find the **price that should be charged to maximize profit**.

2. Use implicit differentiation to compute the value of $\frac{dy}{dx}$ at $(x, y) = (3, 2)$ if

$$x^2 + y^2 = xy + 7.$$

3. A hailstone (small sphere of ice) is forming in the clouds so that its radius is growing at the rate of 1 millimeter per minute. Find how fast its volume is growing at the moment when its radius is 2 millimeters.

(**Hint:** The volume of a sphere of radius r is given by $V = \frac{4}{3}\pi r^3$.)