EXAM 3 - MATH 251

YOUR NAME:_____

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Read each problem **very carefully** before starting to solve it. Each problem is worth 10 points. It is necessary to show **all** your work. Correct answers without explanations are worth 0 points. GOOD LUCK!!

1. Consider the function

$$f(x, y) = \frac{(x - 1)^2 y}{(x - 1)^4 + y^2}.$$

   (a) Find the domain $\mathcal{D}$ of f.

   (b) Show that $\lim\limits_{(x,y)\to(1,0)} f(x, y)$ does not exist. Please, **show all details**.

2. Find the tangent plane to $f(x, y) = \ln(4x^2 - y^2)$ at the point $(1, 1)$.

3. Use the linear approximation to $f(x, y) = \sqrt{\dfrac{x}{y}}$ at $(9, 4)$ to estimate $\sqrt{\dfrac{9.1}{3.9}}$.

4. Suppose that over a certain region of space the electrical potential V is given by

$$V(x, y, z) = \sqrt{(x + 2y + 3z)^3}.$$

(a) Find the rate of change of the potential at $P = (1, 1, 2)$ in the direction of $v = 2j - k$.

(b) In which direction does V change most rapidly at P?

(c) What is the maximum rate of change at P?

5. Consider the ellipsoid
$$x^2 + 2y^2 + 3z^2 = 1.$$

Find the points on the ellipsoid where the tangent plane is parallel to the plane $3x-y+3z = 1$.