

EXAM 3 - MATH 112

Friday, March 28

YOUR NAME: _____

George Voutsadakis

Read each problem **very carefully** before starting to solve it. Each problem is worth 10 points. It is necessary to show **all** your work. Correct answers without explanations are worth 0 points. GOOD LUCK!!

1. Suppose a continuous function is given.

- You compute the first derivative and find that it has critical points $x = -2$ and $x = 5$ and that $f'(x) > 0$ between the two critical points and $f'(x) < 0$ outside this interval.
 - You compute the second derivative and find that it has a critical point $x = 1$ and that $f''(x) > 0$ if $x < 1$ and $f''(x) < 0$ if $x > 1$.
 - Finally, you find that $f(-2) = -5$, $f(1) = 7$ and $f(5) = 12$.
- (a) Create the joint sign table for the first and second derivatives in order to summarize the information given above.
- (b) Use the table to sketch a possible graph for this function. (Please, try to be neat.)

2. A open-top box with a square base is to have a volume of 4 cubic feet. Find the dimensions of the box that can be constructed using the least amount of material.

(a) Make a sketch on the left and write an equation for the objective function showing clearly the meaning of the variables used.

(b) Write an auxiliary equation and solve for one of the variables.

(c) Return to the objective function and optimize.

3. Right now a peach tree will yield 100 pounds of peaches, selling for 40 cents per pound. Each passing week will increase the yield by 10 pounds, but the selling price will go down by 2 cents per pound. Let t be the number of weeks we wait until harvest.

- (a) Write an equation for the yield per tree as a function of t .

$$Y(t) =$$

- (b) Write an equation for the price per pound as a function of t .

$$p(t) =$$

- (c) Write an equation for the revenue from each tree as a function of t .

$$R(t) =$$

- (d) Find how long a farmer should wait to pick the fruit in order to maximize his revenue.

4. Suppose that y is implicitly defined in terms of x by the equation

$$3x^2 + 7xy^2 + y^3 = -33.$$

- (a) Compute the derivative $\frac{dy}{dx}$.

- (b) Find an equation for the tangent line to the graph of the given equation at the point $(x, y) = (-1, -2)$.

5. A spherical balloon is being deflated so that its volume is decreasing at the rate of 48π cm^3/min . Find the rate at which its radius is decreasing when the radius is 2 cm.
(The volume of a sphere is given by $V = \frac{4}{3}\pi r^3$.)