EXAM 4 - MATH 310   Thursday, April 17

YOUR NAME:_____   George Voutsadakis

Read each problem **very carefully** before starting to solve it. Each problem is worth 10 points. It is necessary to show **all** your work. Correct answers without explanations are worth 0 points. GOOD LUCK!!

1. Solve the initial value problem

$$y'' - 4y' + 9y = u_3(t), \quad y(0) = 0, \quad y'(0) = 1.$$

2. Solve the initial value problem

$$2y'' + y' + 4y = \delta \left(t - \frac{\pi}{6} \right) \sin t, \quad y(0) = 0, \quad y'(0) = 0.$$

(**Hint**: The Laplace transform $\mathcal{L} \left\{ \delta \left(t - \frac{\pi}{6} \right) \sin t \right\}$ is not given in the table.)

3. Compute from scratch the following.

    (a) The Laplace transform of $f(t) = u_7(t)f(t-7)$ if $F(s) = \mathcal{L}\{f(t)\}$ is known.

    (b) The convolution function $(f * g)(t)$, where $f(t) = t$ and $g(t) = e^{2t}$.

4. Solve the following system of linear differential equations.

$$\mathbf{y}' = \begin{pmatrix} 1 & -4 \\ -2 & 3 \end{pmatrix} \mathbf{y}.$$

5. Solve the following system of linear differential equations.

$$\mathbf{y}' = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 1 & 1 \end{pmatrix} \mathbf{y}$$

and leave your answers in real form.